Inequality, Referral Networks, and Occupational Segregation

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Abstract

This paper studies the impact of racially segregated referral networks on inequality and aggregate welfare. I show that there are racial differences in the composition of referral networks and the use of referral networks by occupation. In particular, non-college black and white workers in the United States who obtain a job via referral display substantial social segregation, using same-race contacts around 90% of the time. While non-college black and white workers use referrals at a similar rate overall, black workers use referrals for higher-skill and higher-paying occupations at a lower rate than white workers. I also document racial differences in occupational choice, with white workers sorting into higher-skill occupations. I connect and rationalize these observations by incorporating a referral-based matching function into a standard search and match model with occupational choice, heterogeneous ability levels, free entry, and wages determined by Nash bargaining. Social segregation can lead to differences in occupational choice by race, and thus wage and employment inequality, in the steady state. After calibrating the model to examine black and white workers in the United States, the estimates show that racially biased networks alone can generate a black-white wage gap of 1.66% and an employment gap of 0.74 percentage points. Moving from the segregated to the desegregated steady state harms the majority white workers while helping the minority black workers, resulting in a decrease in aggregate welfare.

1 Introduction

To what extent can racially segregated social networks perpetuate inequality? This paper focuses on differences in access to vacancy information by race and occupation through a workers' referral network. Previous research has established the importance of networks in the hiring process, with studies finding that approximately half of all Americans use social contacts to find jobs (Rees 1966; Granovetter 1995; Topa 2001). These connections can provide job seekers with information about employment opportunities and improve their chances of securing a position. Studies have also shown that social networks are racially segregated, i.e. homophilous; in a national probability sample, only 8% of adults reported having a person of another race "with whom they discuss important matters" (Marsden 1987). More recent evidence on various homophilous social networks is given by Mayer and Puller (2008), Currarini et al (2009), or Zeltzer (2020). Even if employers lack discriminatory intent, the use of referrals combined with racially structured networks of friends and families is not race-neutral. Workers' and firms' reliance on racially homophilous networks to facilitate matches can generate inequalities in the type of job opportunities available to black and white workers. In this paper, I will argue that social divisions combined with employment divisions can generate a feedback loop that reinforces a stable path for occupational segregation by race.

I establish several key empirical facts. First, using the Current Population Survey (CPS) for the years 1995-2021 I show that occupational segregation for black and white non-college workers has been relatively stable for the previous two decades[1]. I then delve into an explanation for occupational segregation that is under-explored using the Survey of Consumer Expectations (SCE) Job Search supplement, available for 2013-2021. I show that non-college black and white workers use family/friend referrals at a similar rate to find jobs

[1] Previous research has also documented that occupations segregated by gender, racial and ethnic groups are aligned along stable segregation paths. See Padavic and Reskin (2002), England et al (2020), and Weeden et al (2019).

overall, but the use of referrals differ by occupation. More precisely, examining referral usage along this new dimension suggests that black workers have relatively better access to lower skill and lower paying jobs via referral networks, while white workers have significantly better access to higher skill and higher paying jobs via referral networks. Finally, I use the Multi-City Study of Urban Inequality (MCSUI), available for the years 1992-94, to look at the rate at which workers find jobs through same-race referrals by occupation. I find substantial racial homophily; both black and white workers rely on same-race contacts approximately 90% of the time. However, the extent of this homophily differs by occupation. Both black and white workers rely more heavily on white contacts for higher skill and higher paying jobs, and black contacts for lower skill and lower paying jobs.

I then develop a model to rationalize these patterns in the data. The model does not include any discrimination by employers and does not rely on any heterogeneity between racial groups beyond their networks to generate racial differences in outcomes. I embed a matching function that incorporates a referral network with racial homophily into a standard search and match model with multiple occupations, heterogeneous ability levels, free entry, and wages determined by Nash bargaining. Unemployed workers must choose which occupation they want to search in. The more friends a worker has who are employed in the occupation they are searching for a job in, the likelier they are to hear about an open vacancy and become employed. However, the occupation that fully exploits an individuals' referral network may not be the same occupation that fully exploits their productive advantage. This generates a possible trade-off between the rate at which an individual finds a match and the productivity of that potential match. I am able to derive group differences endogenously even when groups share identical fundamentals, with initial differences in access to vacancy information perpetuated in equilibrium by induced differences in occupational choice between racial groups.

I calibrate the model to estimate the values of the structural parameters required to generate the patterns observed in the data. I am able to match the amount of occupational segregation observed in the data, as well as the rate of referral usage by race and occupation. Under the same set of parameter values, the model also supports a desegregated equilibrium, allowing for counterfactual analysis and policy recommendations. I find that socially segregated networks alone can generate a 1.66% difference in wages between black and white workers and a 0.74 percentage point gap in employment. Welfare analysis reveals that moving to the desegregated steady state would on average harm white workers while helping black workers. I argue that evaluation of programs such as affirmative action should include network effects.

After a literature review, Section 2 offers empirical motivation for the models' main mechanism. Sections 3-4 introduce and solve the model, highlighting the externality introduced in the matching function via referrals. Section 5 discusses calibration, and Section 6 presents the model results.

1.1 Literature

This paper ties together several ideas, including the role of referral networks in the labor market, occupational mismatch, and occupational segregation. Granovetter's influential 1973 paper, published in the Journal of Sociology, established the foundation for network research in the social sciences. He emphasizes that employment information is intertwined with routine social interactions, and differences in the sources of information people access when finding work can affect the type of jobs they obtain. Loury (1976) is another early paper discussing racial inequality and introducing the idea of social capital. "It is widely held that the elimination of racial discrimination will result in the eventual elimination of economic inequality". But Loury argues that such a view does not adequately take into account the importance of parental economic success, and its effect on offspring's opportunities to acquire skills. Since then, there have been a number of papers that focus on the relationship between referral networks and inequality. Key roles of social networks as a driver of inequality are discussed in DiMaggio and Garip (2012), Small (2009), DiMaggio and Garip (2011), and the afterward of the second edition of Granovetter (1995). DiMaggio and Garip (2012) give an overview of potential network mechanisms that can generate inequality, emphasizing that networks can exacerbate inequality when individual differences are compounded by social networks. Other papers include Braddock and McPartland (1987), who use an indirect measure (the racial composition of the respondent's high school) to observe that Blacks who are embedded in racially segregated networks have lower incomes and are much less likely to have White coworkers than their counterparts who are embedded in racially heterogeneous networks. They conclude that racially homogenous networks disadvantage Blacks in the labor market because they contain less beneficial information about jobs, and my results complement the conclusions drawn in their research. Kugler (2003) can explain some wage inequality among equally productive workers using referrals, which is also in line with the results of my research. Beaman et al. (2018) present evidence that the use of referrals reinforces unequal access to jobs between men and women.

While the primary emphasis of my paper is on race, the analytic framework outlined herein could be applied to other social groups as well, including gender. A recent paper by Chetty et al. (2022) found that two thirds of differences in upward economic mobility across communities can be accounted for by differences in a measure of social capital that captures the amount of homophily by socio-economic status. Topa (2001) uses neighborhood interactions in job search to explain the concentration of unemployment across neighborhoods in Chicago. These empirical results are consistent with theoretical results of Calvo-Armengol and Jackson (2004), who develop a model where agents gather job information through their networks. I am able to generate higher unemployment within one group using a different mechanism, where a particular group of workers rationally select into an occupation with a higher separation rate.

More recently, Galenianos (2021) examines hierarchical referral networks. In this paper, there are type-A workers who have a higher probability of forming a match when meeting a firm and potentially higher productivity on the job than type-B workers. This leads to workers of both types having the majority of their links with type-A workers and type-A workers benefit the most from the use of referrals. This type of structure then exacerbates the already existing inequality between type-A and type-B workers. This contrasts with the literature's usual assumption of an exogenously given and homophilous network structure (an assumption I maintain in this paper).

There is a small literature examining social networks as a mechanism for group educational and occupational segregation, including Buhai and van der Leij (2023) and Pothier (2018). Buhai and van der Leij show that occupational segregation can be supported in an equilibrium when individuals are more likely to form with-in group ties. They examine segregation in terms of educational choices, leading to differences in occupation, as a result of social color homophily via a static four-stage partial-equilibrium model where workers choose their education in the first stage. They then form network ties, followed by the job search process and finally earning a wage and consumption. Their model results lead to complete segregation, i.e. at least one social color choosing only one educational path/occupation. They also find that the segregated equilibrium is always socially optimal. These results could be driven by not allowing for mismatch between a workers educational choices and their innate ability. In contrast, the model in this paper supports partial segregation and incorporates a notion of mismatch that allows for the possibility that the desegregated steady state admits higher aggregate welfare, although I do not find this to be the case. Pothier also focuses on human capital investments and focuses on the allocative implications of segregation. The model incorporates a market with asymmetric information and worker-specific skill type that effects the costs to specializing in a particular occupation. A key component of Pothier's model is the firm's inability to observe the workers' skill-type, so that wage contracts cannot be written contingent on a workers' productivity. This generates another externality where workers do not internalize the decreased productivity generated by a mis-allocation of talent.

There are several single-firm case studies that find that the racial composition of a firm's referral hires tends to reflect the racial composition of the firm's incumbent employees (Fernandez et al., 2000; Petersen et al., 2000; Fernandez and Sosa, 2005). Rubineau and Fernandez (2010) rethink the question of how referrals contribute to job segregation by analyzing existing datasets from the referrer's perspective, rather than the one referred. In their focus on jobs such as janitor, machine operator, sales worker, cashier, and secretary, the authors conclude that reliance on referrals need not generate homophily and continued segregation, so long as conscious efforts are made to increase the volume of referrals by under-represented groups.

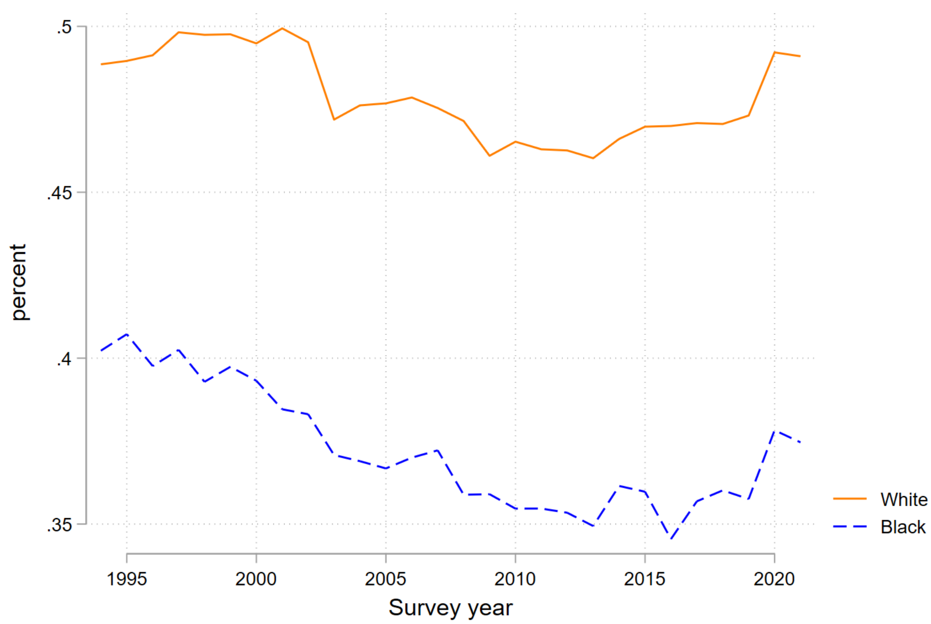
The behaviors generated by the model in this paper can also be related to the literature on statistical discrimination. The statistical discrimination literature similarly derives group differences endogenously even when groups share identical fundamentals, and tends to generate multiple equilibria. Statistical discrimination models rely on the idea of "self-fulfilling prophecies", which refers to when an employers adverse prior beliefs about a group's skill levels are self-confirming in equilibrium. While the statistical discrimination literature generates these patterns using differences in beliefs, this model is able to generate these patterns using differences in access to vacancy information. Initial differences in access to vacancy information are perpetuated in equilibrium by induced differences in occupational choice between racial groups.

2 Empirical Findings

I first document an empirical finding that is not novel–that black and white workers are segregated by occupation, with black workers segregated into less productive occupations. I then document some novel empirical findings: the rate at which black and white individuals use social contacts is similar, but differ substantially within occupation. In particular, black workers are more likely to obtain a job through a social contact for low-paying occupations, while white workers are more likely to obtain a job through social contacts for higher-paying occupations. Further, I find that workers rely on same-race social contacts heavily regardless of occupation, but both black and white workers rely more heavily on black contacts for low skill jobs and white contacts for high skill jobs.

Occupational segregation and wage data is measured using the Current Population Survey (CPS) for the years 1995-2021, and is used to show a persistent gap in wages and occupational choice between black and white workers. IPUMS-CPS micro-data is a nationally-representative monthly sample of the US population with data starting in 1976. I'm limiting the scope of the analysis to individuals who do not have a college-degree. A recent paper by Lester, Rivers, and Topa (2021) show that referrals from business contacts primarily help high productivity workers with high incomes and a high skill level, while referrals from family and friends provide a key source of information for workers at the lower end of the income distribution, particularly in low skill markets. The model will rely on an exogenously determined network structure, which is less relevant in the world of high-skill workers where contacts are formed on the job. By focusing on low-skill workers, this simplification better reflects reality. Occupations are ranked by skill level, which is approximated by the mean real hourly wage of non-college prime-age (ages 25-54) workers in each occupation at the two-digit SOC level[2]. I've classified occupations as either high skill (H) or low skill (L) so that about half of non-college workers are employed in each. This is the categorization I'll be working with in my model. While working with a larger number of occupations would be informative, limited observations by race in the SCE and MCSUI prevent a more disaggregated analysis. Figure 1 shows the gap between the percentage of employed black and white workers in H occupations, and you can also see the lack of decline in segregation over the past 2 decades.

Figure 1: Percent Employed in H Occupation (non-college workers)



In order to see that this segregation cannot be fully explained by differences in the attributes of white and black workers, I want to examine segregation conditional on a set of relevant covariates. To do this, I will construct counterfactual employment distributions in which black workers are given the characteristics of white workers (that are observable in the CPS) using the methodology laid out in Gradin (2013). See Appendix A for the full details of this decomposition technique. I find that about half of the differences in occupational choice

[2]Two-digit SOC is used to maintain a consistent classification of occupations between datasets.

can be accounted for by observable factors, including age, education, marital status, gender, and state. The raw percentage of black workers employed in H-type occupations average over 1995-2021 is 37% and for white workers is 48%, while the conditional percentage of black workers employed in H-type occupations is 43%. The 11 percentage point gap closes by 6 percentage points, just over half.

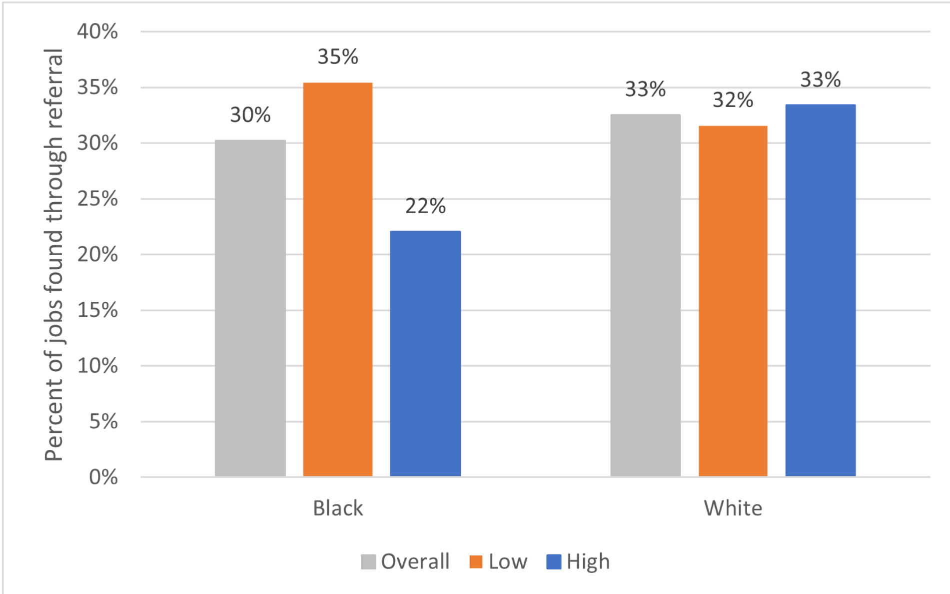
Next, I want to examine if black and white workers have differential access to these occupations through their friend networks. Some of the most direct evidence available is through the Survey of Consumer Expectations Job Search supplement, which is a survey fielded annually each October since 2013 that focuses on job search behavior and outcomes for all individuals, regardless of their labor force status. Existing labor force surveys typically only collect information on the search behavior of the unemployed. The survey asks an expansive list of questions on the employment status and current job search, if any, of all respondents, including questions on an individual's search effort, search methods and outcomes, and the incidence of informal recruiting methods. Demographic data is also available for respondents[3]. Note that this sample is a set of annual repeated cross-sections. The main monthly SCE surveys its respondents for up to 12 months. Since the Job Search Supplement draws from these respondents each October, individuals will be in the supplement only once. Here I present estimates for a sample that pools the 2013–2021 data together.

Figure 2 shows the percentage of jobs obtained by race and occupation-type. It suggests that black and white workers successfully use their social networks to obtain a job at a similar rate, but there's a difference in access when it comes to the occupation type. Black workers are able to access low-paying jobs through their network at a slightly higher rate than white workers, and white workers are able to access higher-paying jobs through

[3] In Faberman, Mueller, Sahin, and Topa (2022), they show that overall and for each survey year, the Job Search Supplement matches the demographics of the CPS reasonably well, with some exceptions being that the Job Search Supplement contains a higher shares of White, older, and married individuals compared to the CPS.

their network at a substantially higher rate than black workers.

Figure 2: Referral Usage by Race and Occupation



To establish that this relationship is not just capturing worker characteristics, I model the determinants of employed workers having found a job through a referral. I want to examine, conditional on being employed, the probability of using friends and relatives to obtain a job. The probability of success is assumed to be a logistic function where $z = 1$ if an employed worker finds a job through a friend or relative referral, and $z = 0$ if the found the job in some other way. I run a worker-level logistic regression on a dummy variable for referral usage against an interaction between indicator variables for being white and occupation type. I also control for state fixed effects, and a set of worker characteristics including education, age, gender, union status, and marital status. Then

$$P(z_i = 1 \mid e_i = 1) = \frac{exp(\beta' x)}{1 + exp(\beta' x)}$$

where x is my set of explanatory variables and e_i is an indicator of employment status. Let

p be the probability that a referral is used. Then

$$\text{logit}(p) = \ln\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1\text{H} + \beta_2\text{white} + \beta_3(\text{H} \times \text{white}) + \text{controls}$$

Table 1: Regression for use of referrals

H occupation	-0.663	-0.853	-0.979*
	(.463)	(.500)	(.531)
White dummy	-0.174	-0.375	-.407
	(.327)	(.365)	(.374)
Interaction	0.746	1.184**	1.327**
	(.501)	(.580)	(.623)
State FE			x
Worker Characteristics		x	x
Psuedo R2	0.003	0.090	0.16
Obs	866	744	733

Robust standard errors are shown in parenthesis.
*$p < 0.1$; **$p < 0.05$

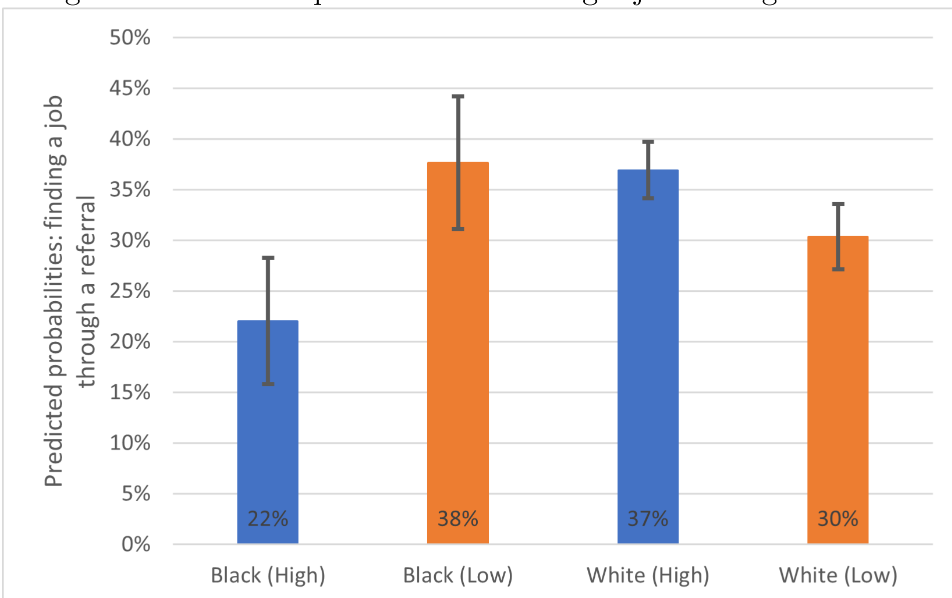
Results, which are reported in Table 1, are robust to excluding union status and to using regional instead of state fixed effects. The positive and significant coefficient on the interaction term tells us how the likelihood of using a referral differs across groups (blacks and whites) by occupation. Here it's positive, indicating that whites are likelier to use their friend network when obtaining H type jobs than their black counterparts. Specifically, I have $\exp(1.327) = 3.77$. This means that a white person in an H-type job has about 3.77 times the odds of the black person using a referral of working in an H job, or equivalently, a 3.77-1 = 2.77, 277% more odds of having obtained their job through a referral.

To make these results more interpretable, I can calculate the probability that black and white workers obtained their job via a referral by occupation type. Solving for p:

$$p = \frac{exp(\beta_0 + \beta' x)}{1 + exp(\beta_0 + \beta' x)}$$

For each individual, I calculate p using the individuals observed values for all variables except occupation and race. For occupation and race, I calculate the probability of success for all combinations of occupation $j \in \{H, L\}$ and race $r \in \{B, W\}$ for each observation. Then I average the estimated probabilities across all observations for each race/occupation combination, and plot these results (with 90 percent confidence intervals) in Figure 3. These

Figure 3: Predicted probabilities: finding a job through a referral



numbers are quite similar to what we see in the raw data, although the differences between black and white workers are larger once we control for differences in worker characteristics and location. See Appendix A for results examining different occupational groupings.

Not many surveys gather information on the race of the individual offering the referral. Fortunately, the Multi-City Study of Urban Inequality (MCSUI), conducted in 1992-1994, collects this information for individuals in three cities: Atlanta, Boston, and Los Angeles (the survey also includes Detroit, but doesn't ask the specific jobs search questions needed for the analysis below and so this city is excluded). This study is particularly well suited for questions on race because areas with a high proportion of African American residents were oversampled. Table 2 presents the percent of same-race contacts used by black and white workers by occupation, and overall. These results are tabulated using only black or white contacts, and not some third race, although these do make up a small percentage of the contacts.

Table 2: Racial breakdown of references

		same race	other race
White	High	97.0	3.0
	Low	86.0	14.0
	Overall	91.0	9.0
Black	High	81.1	18.9
	Low	95.0	5.0
	Overall	89.0	11.0

Data source: Multi-City Survey of Urban Inequality (MCSUI), author's calculations

Consider the overall rate at which individuals use same-race contacts (i.e. not by occupation). Whites contact whites 91% of the time, while blacks contact whites 11% of the time. For comparative purposes, if these contact rates reflected only the population distribution in this sample, white workers would be contacted 75% of the time. This data supports the main mechanism of this paper's model–that individuals will, for both occupations, get referral information through same-race individuals.

See Appendix A for separate reports by city with statistics that include the use of referral from workers that are not black or white. Appendix A additionally includes the summary statistics and regressions previously generated using the SCE utilizing the MCSUI instead. The same patterns emerge, although only for men.

3 A Model of the Labor Market

The model is designed to rationalize persistent differences by race in occupational choice through their social network. Time is continuous and the labor market is in steady state.

Workers: There is a measure one of infinitely lived workers who are heterogeneous in ability and race. They are either employed in occupation $j \in \{H, L\}$ or unemployed and searching in occupation j[4]. I consider two races $r \in \{W, B\}$. The relative population sizes of each race are denoted by $\tau^r \in [0, 1]$, with $\sum_r \tau^r = 1$. Ability types are indexed by x, and are permanent and observable. Denote $G(\cdot)$ the distribution of ability with corresponding density $g(\cdot)$. Note that the distribution of ability is the same for each group. Consequently, any occupational segregation observed in equilibrium will be a result of strategic decisions made by workers, rather than an assumed disparity in productivity among individuals from different social groups. Each worker is endowed with 1 unit of labor. There is no on-the-job search. Workers have risk neutral preferences and discount the future at rate $\rho \in (0, 1)$.

Firms: A free entry condition determines the measure of firms. These firms either produce output $y_j(x)$, $j \in \{H, L\}$, when matched with a worker of ability x, or post vacancy v_j for workers of any type. Employers are risk neutral and also discount at rate ρ.

Production Tech: There are two types of production technologies that define two types of occupations, and their outputs are perfect substitutes. Technology used at low-skill occupations is not a function of worker ability. Technology is never a function of worker

[4]Using occupations to define separate labor markets is generally a relevant search criterion for both workers and firms. Usually, firms post vacancies for certain qualifications in terms of occupation or education, and workers primarily look for jobs in their occupation. Evidence shows that occupational mobility is generally quite low, such a matching process is in line with this.

race. Specifically,

$$y_j(x) = \begin{cases} A_L & \text{if } j = L \\ A_H x & \text{if } j = H \end{cases}$$

Matching Technology: Every worker is linked with a measure of other workers. Assume the size of each worker's network is the same. A worker's employment opportunities in a particular occupation will depend on how many of their links are employed in that occupation. Having a continuum of links means that the employment rate of a worker's social contacts reflects the aggregate due to the law of large numbers.

The rate at which workers receive information from someone of the same race versus the other race depends on two exogenous parameters. These include in-group bias $\gamma^r \in [0,1]$ and population share $\tau^r \in (0,1)$. When $\gamma^r = 1$, we are operating in a world of complete homophily or social segregation, with workers only receiving information from individual's of the same race. When $\gamma^r = 0.5$, there is no bias and worker's receive information from each race at a rate proportional to their population size τ^r[5]. I generally won't be interested in $\gamma^r < 0.5$. Then the expected share of same-race contacts is

$$\phi^r = \frac{\tau^r \gamma^r}{\tau^r \gamma^r + \tau^{\neg r}(1 - \gamma^r)}$$

The rate at which workers hear about occupation-specific vacancy information depend on ϕ^r and a set of endogenous variables, occupational choice for both black and white workers and the occupation-specific employment rates of these groups. Allow a_j^r to denote the percent (or allocation) of individuals of race r that are searching or working in j oc-

[5] For example, if $\gamma^r = 0.5$ and 75% of the population is white, both black and white workers would expect to get information from white individuals 75% of the time.

cupation. The percent of employed ($\bar{e}^r_j$) and unemployed ($\bar{u}^r_j$) contacts either working or searching in occupation j for an individual of race r are:

$$\bar{e}^r_j = \phi^r a^r_j e^r_j + (1-\phi^r) a^{\neg r}_j e^{\neg r}_j, \quad \bar{u}^r_j = \phi^r a^r_j u^r_j + (1-\phi^r) a^{\neg r}_j u^{\neg r}_j$$

where $\neg r$ denotes the 'other' race. These are simply weighted averages of the endogenous employment rates by race and occupation (e^r_j) and the endogenously determined allocation of individuals working/searching in occupation j by race (a^r_j), where the weights are determined by the exogenous given homophily parameter (γ^r) and population size (τ^r).

Vacancy creation occurs in two ways: a new firm enters the market (creating a standard vacancy), or an already existing firm employing a worker expands at rate κ_j. If a match is formed, the firm immediately sells the position off so that each firm maintains employment of only one worker. As in Galenianos (2014), expansion can be understood in two ways: an entrepreneur partners with an existing firm to fill a new role, or the firm identifies a profitable opportunity but sells the new position due to decreasing returns.

When an expansion occurs, one of the links of the incumbent worker is contacted at random. If the link is unemployed and searching in occupation j then he is hired by the firm and begins work next period; if the link is employed in occupation j, he can potentially receive the vacancy information and instantaneously pass it along to one of his own contacts; otherwise the referral opportunity is lost and it becomes a market vacancy next period. The rate at which the unemployed worker is referred to a job is

$$p^r_j \equiv \kappa_j (\bar{e}^r_j)^{\Gamma_j}$$

The number of meetings is determined by the effective number of employed contacts ($\bar{e}^r_j$), the expansion rate (κ_j), and a measure of nonlinear information dispersion (Γ_j). Cappellari and Tatsiramos (2015) find some evidence of potential convex/nonlinear network effects when looking at the relationship between the number of employed workers in an individual's friend network and the job finding rate[6]. Allowing for $\Gamma_j \neq 1$ takes into account potential non-linearities in this relationship. Note that this referral rate does not depend on the worker's ability level x. Then the flow of meetings via referral by race and occupation is:

$$P_j^r = u_j^r p_j^r$$

Note that we can also define the job filling rate through referrals as the flow meeting of referrals divided by the total number of referral vacancies:

$$k_j^r = \frac{u_j^r (\bar{e}_j^r)^{\Gamma_j}}{a_j^r e_j^r + a^{\neg r_j} e_j^{\neg r}}$$

Incorporating $\bar{e}_j^r$ into the matching function generates an externality when workers chooses to work and search in occupation j, since doing so increases the likelihood that all other workers (and in particular those of race r) searching in occupation j can hear about vacancy information through their network. This externality is the key source of the multiplicity of equilibria in this model. In particular, we have strategic complements–each agent is more willing to take an action (searching in j) when other agents are doing so.

For $\Gamma_j \neq 1$, the relationship between the number of employed contacts you have in an industry and the likelihood of hearing about vacancy information is nonlinear, more in line with the behavior of the network matching function derived in Calvo-Armengol and Zenou (2005). When $\Gamma_j > 1$, there are increasing returns to the relevant employed network size. This concept refers to the idea that output proportionally increases more than input as

[6] In particular, they find that having one employed friend increases the job finding probability by 1.6 p.p., having two employed friends increases it by 4.9 p.p., and having three increases it by 11.1 p.p.

positive feedback mechanisms are triggered. As the player ahead moves further, the player that is behind, in turn, loses further advantages. Increasing returns is an important component found across various phenomena that past studies have used to explain the diffusion pattern of some technologies. For example, direct and indirect network effects (Farrell and Saloner, 1985; David, 1985; Katz and Shapiro, 1986), self-fulfilling expectations (Besen and Farrell, 1994) and learning effects (Dobusch and Schüßler, 2012; David, 1985).

Proposition 4.1. The referral component of the job finding rate p_j^r is decreasing in unemployment u_j^r and $u_j^{\neg r}$ $\forall j, r$, therefore increasing in employment, i.e. $\frac{\partial p_j^r}{\partial e_j^r} > 0$ and $\frac{\partial p_j^r}{\partial e_j^{\neg r}} > 0 \ \ \forall j, r$. Assuming Γ_j is not too large, the referral job filling rate k_j^r is increasing in unemployment u_j^r and $u_j^{\neg r}$ $\forall j, r,$, therefore decreasing in employment, i.e. $\frac{\partial k_j^r}{\partial e_j^r} < 0$ and $\frac{\partial k_j^r}{\partial e_j^{\neg r}} < 0 \ \ \forall j, r$.

Proof. See Appendix B.

Let v_j denote the number of vacancies in occupation j. Vacancies are not targeted to a particular ability type x or race r. The flow of meetings in the market between a vacancy in industry j and a worker of race r searching in industry j is determined by a Cobb-Douglas matching function:

$$M_j = \theta_j (v_j)^{\eta} (\tilde{u}_j)^{(1-\eta)}$$
$$\tilde{u}_j = \sum_r a_j^r \tau^r u_j^r$$

with $\theta_j > 0$ and $\eta \in (0, 1)$ The market job finding rate f_j and market vacancy filling rate q_j

are:

$$f_j = \frac{M_j}{\tilde{u}_j} = \theta_j(v_j)^{\eta}(\tilde{u}_j)^{-\eta} \quad , \quad q_j = \frac{M_j}{v_j} = \theta_j(v_j)^{\eta-1}(\tilde{u}_j)^{1-\eta}$$

The matching function is then given by

$$m_j^r = M_j + P_j^r$$

Proposition 4.2. The aggregate matching function exhibits decreasing returns to scale.

Proof. See Appendix B.

Intuitively, if we consider a proportional increase in the number of unemployed and the number of vacancies, labor market tightness remains the same and so the standard Cobb-Douglas component of the matching function maintains the same level of efficiency. However, the number of employed links offering referrals is lowered and so the meeting rate through referrals is lowered. This corresponds to a decrease in the efficiency of the referral portion of the matching function, and therefore the aggregate matching function.

4 Steady State Equilibrium

Occupational choice will obey a threshold rule: if a worker with ability x finds it optimal to search and work in occupation H, then so will workers with a higher ability level. Then there exists thresholds x_r^* such that for all workers from group $r \in \{B, W\}$ with $x > x_r^*$, they search in occupation H, and for all workers with $x \leq x_r^*$, they search in occupation L. Define the following sets: $x_H^r = \{x | x > x_r^*\}$ and $x_L^r = \{x | x \leq x_r^*\}$. At times, it is only necessary to keep track of the portion of the population of race r searching in H or L (but not their ability type), previously denoted as a_j^r, which can be defined by integrating over density $g(\cdot)$ as $a_j^r = \int_{x_j^r} g(x)dx$.

Let E_j^r denote the value of expansion for a firm in occupation j with a worker of race r. Expansions occur at rate κ_j, and are sold off with incumbent firms receiving share $\alpha \in [0, 1]$ of the value. Either a match is made through the network immediately, or the referral process fails and search through the market begins.

$$E_j^r = V_j + \frac{\phi^r u_j^r k_j^r}{\bar{u}_j^r} \int_{x_j^r} \Big[(J_j^r(x) - V_j)\Big] g(x)dx + \frac{(1-\phi^r) u_j^{\neg r} k_j^{\neg r}}{\bar{u}_j^r} \int_{x_j^r} \Big[(J_j^{\neg r}(x) - V_j)\Big] g(x)dx$$

where $J_j^r(x)$ is the value of a match to the firm with a worker of race r type x in occupation j, V_j is the value of vacancy in occupation j, and k_j^r is capturing the probability of a match with a race r individual occurring. Finally, from the perspective of the employed worker, $\phi^r a_j^r u_j^r / \bar{u}_j^r$ percent of my contacts are same-race and $(1-\phi^r) a^{\neg r_j} u_j^{\neg r} / \bar{u}_j^r$ percent are the other race (note that a_j^r is captured by the integral).

Allow $W_j^r(x)$ denote the value of a match to a worker, and $U_j^r(x)$ denote the value of unemployed search to a worker of race r type x in occupation j. The following value functions define the worker and firm problems:

$$\rho V_j = -c_j + \sum_r \frac{\tau^r u_j^r}{\tilde{u}_j} \int_{x_j^r} \Big[q_j(J_j^r(x) - V_j)\Big] g(x)dx \tag{3}$$

$$\rho J_j^r(x) = y_j(x) + \kappa_j \alpha E_j^r - w_j^r(x) + s_j \Big[V_j - J_j^r(x)\Big] \tag{4}$$

$$\rho W_j^r(x) = w_j^r(x) + s_j \Big[U_j^r(x) - W_j^r(x)\Big] \tag{5}$$

$$\rho U_j^r(x) = z_j + (f_j + p_j^r) \Big[W_j^r(x) - U_j^r(x)\Big] \tag{6}$$

where

$$U_j^r(x) = \max\Big\{U_L^r(x), U_H^r(x)\Big\} \tag{7}$$

where c_j is the vacancy opening cost in occupation j, s_j is the rate at which a match in occupation j is destroyed, and z_j is the flow value of unemployment.

The assumption that search through referral is costless for firms implicitly assumes that the incumbent worker who offers the referral resolves uncertainty about the match quality. Thus the firm doesn't have to go through costly hiring activities, such as advertising vacancies, interviewing, and screening. This mirrors the findings of Montgomery (1991), who demonstrated that profit-maximizing firms can exploit social networks to screen job applicants without incurring costs.

Define match surplus as $S_j^r(x) = J_j^r(x) + W_j^r(x) - U_j^r(x) - V_j$. With Nash bargaining, free entry, and worker bargaining power equal to β, I solve for

$$w_j^r(x) = \arg\max_{w_j^r(x)} (W_j^r(x) - U_j^r(x))^{\beta} (J_j^r(x))^{1-\beta}$$

The Nash bargaining solution along with free entry and my surplus equation gives

$$W_j^r(x) - U_j^r(x) = \beta S_j^r(x) \tag{8}$$

$$J_j^r(x) = (1-\beta) S_j^r(x) \tag{9}$$

Free entry implies $V_j(x) = 0 \ \ \forall j, x$, so that the value of an expansion to a firm can be written as

$$\begin{aligned} E_j^r &= \frac{\phi^r u_j^r k_j^r}{\bar{u}_j^r} \int_{x_j^r} J_j^r(x) g(x) dx + \frac{(1-\phi^r) u_j^{\neg r} k_j^{\neg r}}{\bar{u}_j^r} \int_{x_j^r} J_j^{\neg r}(x) g(x) dx \\ &= \frac{\phi^r u_j^r k_j^r}{\bar{u}_j^r} \int_{x_j^r} (1-\beta) S_j^r(x) g(x) dx + \frac{(1-\phi^r) u_j^{\neg r} k_j^{\neg r}}{\bar{u}_j^r} \int_{x_j^r} (1-\beta) S_j^{\neg r}(x) g(x) dx \\ &\equiv \bar{S}_j^r \end{aligned}$$

Free entry also gives us

$$
\begin{aligned}
c_j &= \sum_r \int_{x_j^r} \Big[q_j^r J_j^r(x) \Big] g(x) dx \\
&= \sum_r \int_{x_j^r} \Big[q_j^r (1-\beta) S_j^r(x) \Big] g(x) dx
\end{aligned} \tag{10}
$$

The value function for surplus is:

$$
\rho S_j^r(x) = y_j(x) - z_j - S_j^r(x)(s_j + \beta(f_j + p_j^r)) + \alpha \kappa_j \bar{S}_j^r \tag{11}
$$

An existing match generates $y_j(x)$ units of output. If the match separates, the value of the firm falls to zero and the worker becomes unemployed, getting z_j. They become employed again at rate $f_j + p_j^r$ and keep a share β of the match surplus. At rate $(1 - s_j)$, the match is maintained next period. Finally, there's the surplus generated by the potential expansion and use of the current worker's network. From eq (4), we have

$$
(\rho + s_j)(1-\beta) S_j^r(x) = y_j(x) - w_j^r(x) + \alpha \kappa_j \bar{S}_j^r
$$

Solving for wages yields:

$$
w_j^r(x) = y_j(x) - (1-\beta)(s_j + \rho) S_j^r(x) + \alpha \kappa_j \bar{S}_j^r \tag{12}
$$

I will derive four expressions summarizing the steady state equilibrium: the job creation curve, the wage equation, the unemployment equation, and a condition representing how agents chose whether to search for low or high skilled occupation.

Steady state surplus equation:

$$S_j^r(x) = \frac{y_j(x) - z_j + \alpha\kappa_j \bar{S}_j^r}{\rho + s_j + \beta(f_j + p_j^r)} \tag{13}$$

Job Creation Curve: the free entry condition, along with (3) and (4), yield the job creation curve:

$$c_j = \sum_r \frac{\tau^r u_j^r}{\tilde{u}_j} \int_{x_j^r} \left(q_j \frac{y_j(x) - w_j^r(x) + \alpha\kappa_j \bar{S}_j^r}{\rho + s_j} \right) g(x)dx \tag{14}$$

Steady State Wages: Plugging my steady state equation for surplus into (12) yields

$$w_j^r(x) = y_j(x) + \alpha\kappa_j \bar{S}_j^r - (1-\beta)(s_j + \rho)\left[\frac{y_j(x) - z_j + \alpha\kappa_j \bar{S}_j^r}{\rho + s_j + \beta(f_j + p_j^r)}\right] \tag{15}$$

Steady State Employment: The steady state conditions for flow into and out of unemployment and employment, conditional on a worker searching/working in occupation j, are:

$$u_j^r = \frac{s_j}{f_j + p_j^r + s_j} \tag{16}$$

$$e_j^r = \frac{f_j + p_j^r}{f_j + p_j^r + s_j} \tag{17}$$

Note that since the job finding rate through both the market and the network does not depend on your ability type x, employment rates will be the same for all x within each race and occupation.

Occupational Choice When unemployed, workers endogenously choose whether to search for high or low skilled occupations. They make this decision by maximizing over the future discounted value of both options. In steady state, this decision, laid out in equations (6) and (7), (with substitution from eq (5)), gives me:

$$\rho U_j^r(x) = \frac{(f_j + p_j^r)w_j^r(x) + (\rho + s_j)z_j}{\rho + f_j + p_j^r + s_j}$$

So that the choice becomes

$$\max_j \rho U_j^r(x) = \max_j \left[\frac{(f_j + p_j^r)w_j^r(x) + (\rho + s_j)z_j}{\rho + f_j + p_j^r + s_j}\right] \tag{18}$$

This choice function includes both individual endowments and the extent to which one's network have already chosen a particular occupation. Equations (14), (15), (16), and (18) then determine the steady state equilibrium.

5 Calibration

In this section I calibrate the model to uncover the values of the structural parameters required to generate the patterns we observe in the data. Table 3 lists the parameter estimates. The following section provides details on how these parameters were estimated.

Occupation categorization: Occupations group jobs based on the task or skill content of their employees, while industries group jobs based on the product category of their output. This distinction makes occupations a better dimension along which to divide vacancies into low- and high-skilled. Occupations are ranked by skill level, which is approx-

Table 3: Calibration

Parameter	Value	Source	Description
Taken from the literature:			
β	0.50		worker bargaining power
ρ	0.0033	(monthly)	discount rate
η	0.565	Galenianos (2014)	elasticity matching function
α	0		expansion surplus share
c_H, c_L	0.13 , 0.10	CPS	vacancy posting cost
s_H, s_L	0.034, 0.043	CPS	separation rate
$\gamma^B,\ \gamma^W$	0.96 , 0.77	MCSUI	homophily rate
$\tau^B,\ \tau^W$	0.25 , 0.75	MCSUI	population share
$z_H,\ z_L$	0.055, 0.208		flow utility of unemp
$A_H,\ A_L$	0.761, 1.254		production tech
$\theta_H,\ \theta_L$	0.042, 0.048		market matching tech
$\kappa_H,\ \kappa_L$	0.444, 0.103		firm expansion rate
$\Gamma_H,\ \Gamma_L$	2.411, 1.176		nonlinear matching tech
σ_x	0.368		sd of ability

imated by the mean log wage of non-college workers in each occupation. See Appendix A for a table of 2-digit occupations and the calculated median and mean wage.

Separation rates: For simplicity I group unemployed and not in the labor force into one group, the non-employed. While these are distinct labor market statuses, Elsby and Shapiro (2012) and Juhn, Murphy, and Topel (1991, 2002) argue the boundary is blurred over the long-run. The unemployed resemble those not in the labor force because they have long spells of joblessness and few employment opportunities. Since I'm focusing on the steady state, it is reasonable to group these individuals together. Beyond this, Fallick and Fleischman (2004) and Hornstein et al. (2014) show that the number of out-of-the labor force individuals who transition to employment is greater than the number of unemployed people who transition to employment in a given month, again indicating that it would be reasonable to treat these individuals as one group. For the separation rates, I calculate flows out of employment and into non-employment by occupation type using IPUM-CPS, and find that $s_L = 0.043$ and $s_H = 0.035$.

Vacancy posting costs: Vacancy costs are meant to capture the cost of recruitment, which is incurred before a match is formed, and encompasses the cost of advertising vacancies, interviewing, and screening. There is evidence that vacancy posting costs vary by skill[7]. Barron, Berger, and Black (1997) use the 1982 Employment Opportunity Pilot Project survey of 5700 employers, and provide evidence for the time and costs involved in recruiting workers. In particular, they estimate that the average labor cost of hiring one worker is 3% to 4.5% of quarterly wages of a new hire. Here I assume it's 3.5%.

Beginning with a redesign of the survey in 1994, three new questions were added in rotation groups 2–4 and 6–8 that asked individuals who reported being employed in the previous month as well as the current month whether they still worked for the same employer (empsame), whether their job activities and duties were the same, and whether the occupation and work activities reported last month were still accurate for the current month. Fallick and Flesichman (2004) were among the first to use these variables to measure labor flow dynamics in the CPS. If someone reports being out of the labor force or unemployed in the initial month but employed in the next month, they are counted as a new hire. Another group of newly hired workers are those who are employed from one month to the next but who switch employers. These hires can be determined by individuals who report that their employers are not the same as in the previous month. Using low-skilled real monthly earnings as the numeraire, I have $c_L = 0.10$ and $c_H = 0.14$.

Bargaining Power: Typically, scholarly literature assigns a value of 0.50 to represent bargaining power between workers and firms so that it is evenly divided between the two actors. For the baseline model, I will also set bargaining power to 0.50.

[7]For example, according to Dube et al. (2010), the estimated replacement costs in California amount to \$2,500 for blue-collar workers and \$8,800 for professional workers (both in 2013 dollars).

Expansion surplus share: In the baseline model I set α, which dictates the share of surplus kept by the expanding firm after a successful expansion, to zero. I will show that when I recalibrate the model assuming that the true value for α is positive, this increases wage inequality. In that sense, the baseline results can be viewed as a conservative estimate for the wage inequality produced by my proposed mechanism.

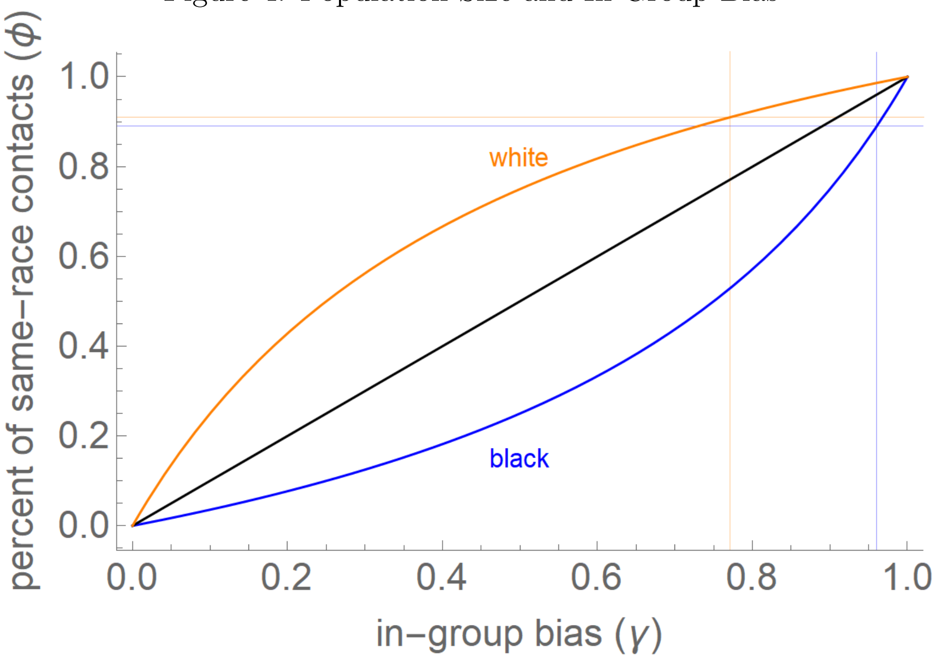
Population Size τ^r and in-group bias γ^r: Using the known values for τ^r from the MCSUI, I can back out values for γ^r that coincide with the rate at which individuals use same-race contacts in the MCSUI data:

$$\frac{\tau^W\gamma^W}{\tau^W\gamma^W + \tau^B(1-\gamma^W)} = .91 \Longrightarrow \gamma^W = .77$$
$$\frac{\tau^B\gamma^B}{\tau^B\gamma^B + \tau^W(1-\gamma^B)} = .89 \Longrightarrow \gamma^B = .96$$

Figure 4 displays how in-group bias γ^r and the percent of same-race contacts ϕ^r vary together, holding τ^r fixed. With $\gamma^r = 0.5$, the probability of using a same-race tie is simply the population size τ^r. Since $\tau^B < \tau^W$, but ϕ^B is very nearly the same as ϕ^W, a larger value for γ^B relative to γ^W is implied.

Ability distribution (x): The remaining parameters are calibrated to fit moments observed in the CPS and SCE. The ability index x is used to characterize the heterogeneity of workers and identify variables related to that heterogeneity (in particular, wages and occupational choice). Nowhere in the model does x appear alone; it does not mean anything by itself, and needs to be interacted with the firm's production technology A_H to have an interpretation. I use a log-normal distribution to model ability and set the mean μ_x to 0.5 arbitrarily, as it will have no effect on the results once A_H is appropriately calibrated. However, the variance of x is important for capturing the wage distribution in the high-type occupation. I use the 90th percentile wages for high-type occupations from the CPS (with

Figure 4: Population Size and In-Group Bias



average low-skill wages as the numeraire) as a calibration target to discipline the value for σ_x.

Matching and production tech ($\kappa_j,\ \Gamma_j,\ \theta_j,\ A_j,\ z_j$): I use these 10 additional parameters to match 10 additional moments from the data. In particular, I match the percentage of black and white individuals employed in high and low occupations, the rate at which black and white individuals use referrals by occupation, as well as job finding rates by race, and average wages by occupation.

Matching efficiency θ_j measures the productivity of the process for matching unemployed workers to open vacancies. Allowing for heterogeneity by occupation helps me capture differences that affect the frictions characterizing the market matching process[8]. I find that the matching efficiency for H occupations is lower than it is for L occupations. Barnichon and Figura (2015) point out that "hiring for high-skill occupations may be more

[8]For example, differences in skill requirements, search channels, search intensity, screening problems, etc.

time-consuming than hiring for low-skill occupations. As a result, low skill occupations may display a higher number of new matches per unit of time (for a given number of job seekers and job openings), i.e., a higher matching efficiency." This is reflected in my calibrated results here.

Unemployment flows z_j measure the value of being unemployed. I find that unemployment flows are lower in H occupations than L occupations, potentially reflecting the higher search costs demanded by H occupations.

6 Results

I am able to hit all of my targeted moments for the segregated steady state, and then examine what the desegregated steady state would look like in a world governed by the same fundamentals[9]. It is important to highlight that the model can generate endogenous differences between two otherwise identical groups[10]. Table 4 displays the targeted moments for the segregated steady state, as well as those moments for the model-generated desegregated steady state[11]. Average wages in the low-type occupation are used as the numeraire.

I can also examine how well the model results match what I see in the MCSUI when broken down by both race and occupation, seen in Table 5. White and black workers rely on contacts with white workers at a higher rate for high-type jobs than the model predicts. This indicates there may be additional bias in the use of contacts by race and occupation

[9]Note that the desegregated equilibrium here is identical in all meaningful ways to the desegregated equilibrium induced by setting the racial bias parameter $\gamma^r = 0.5\ \forall r$. While this would change the weights ϕ^r, these are inconsequential once $a_j^B = a_j^W$ and $u_j^B = u_j^W\ \forall j$.

[10]I allow relative population sizes to differ in the baseline model. However, this asymmetry is not necessary to generate the patterns seen in the data–you do not need a majority/minority group dynamic. See Appendix C for a version of results where $\tau^r = 0.50\ \forall r$ and $\gamma^r = 0.90\ \forall r$.

[11]While the focus is on the observed segregated and theoretical desegregated equilibria, it's interesting to note that there is one more segregate equilibrium supported by the model and this set of parameter values, where black workers are overemployed in H occupations and white workers are overemployed in L occupations. Details of this equilibrium are left to Appendix C.

Table 4: Model Fit and Desegregated Equilibrium

	Data	Targeted	Segregated	Desegregated
% B employed in *H*	37%	Y	37%	45%
% W employed in *H*	48%	Y	48%	45%
job-finding rate (B)	0.10	Y	0.10	.0998
job-finding rate (W)	0.10	Y	0.10	.0998
mean wages (H)	1.39	Y	1.39	1.395
90th percentile wages (H)	2.22	Y	2.22	2.26
mean wages (L)	1.00	Y	1.00	1.001
% jobs from network:				
Black (L):	35%	Y	35%	32%
Black (H):	22%	Y	22%	30%
White (L):	32%	Y	32%	32%
White (H):	33%	Y	33%	30%

Data sources: CPS, SEC, author's calculations

that is not captured by the model.

Table 5: Same-race friend/family references

		Data	Model	Difference
White	High	97.0	92.7	4.3
	Low	86.0	88.9	-2.9
Black	High	81.1	92.8	-11.7
	Low	95.0	95.2	-0.02

Data source: MCSUI, author's calculations

Overall, the model is able to capture differences in occupational choice by race. The model is able to match the targeted moments, including the percentage of black and white individuals employed in high and low occupations, the rate at which black and white individuals use referrals by occupations, as well as job finding rates and wages for high and low-type occupations. I can now measure the model's predicted amount of employment and wage inequality due to network effects. For employment inequality, I find white workers have an employment rate 0.74 percentage points higher and for wages white workers have 1.66% higher wages, which explains about 10% of the difference in employment rates for non-college black and white workers and 14% of the difference in hourly wages for that same group. To get a better feel for the relative importance of this mechanism in explaining wage differentials, I've run linear regression on log hourly wages for non-college black and white workers in the CPS. When controlling for age and its square, education, gender, marital status, as well as state and year fixed effects, I am able to explain 36% of the observed hourly wage differential for this group. Finally adding occupation (using 4 digit codes) and industry controls I am able to explain 62% of the gap.

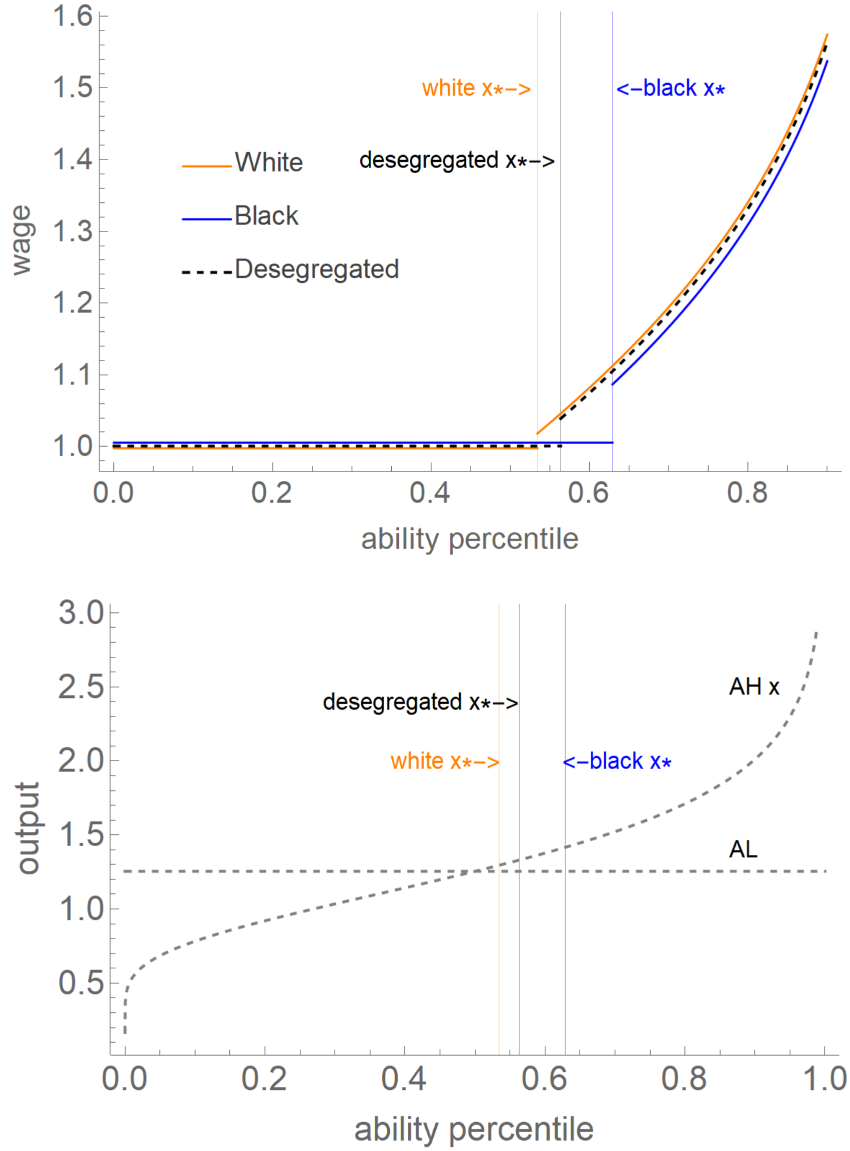
The differences in employment rates are due to black workers selecting into an occupation with a higher separation rate. Table 6 displays employment rates by race and occupation in both the segregated and desegregated steady states. Figure 5 displays the wage profiles of black and white workers in the segregated equilibrium, as well as their (identical) wage profiles in the desegregated equilibrium. Figure 5 also displays the productivity of each ability x worker if they were employed in H or L occupation types, as well as vertical lines at x_r^* in the segregated equilbrium, labeled as 'black' and 'white', and the desegregated equilibrium (where x_r^* is the same for both black and white workers).

Table 6: Employment Rates

	Segregated		Desegregated
	Black	White	Both
Overall	**70.1**	**70.9**	**70.6**
High	69.94	73.04	72.16
Low	70.24	68.98	69.45

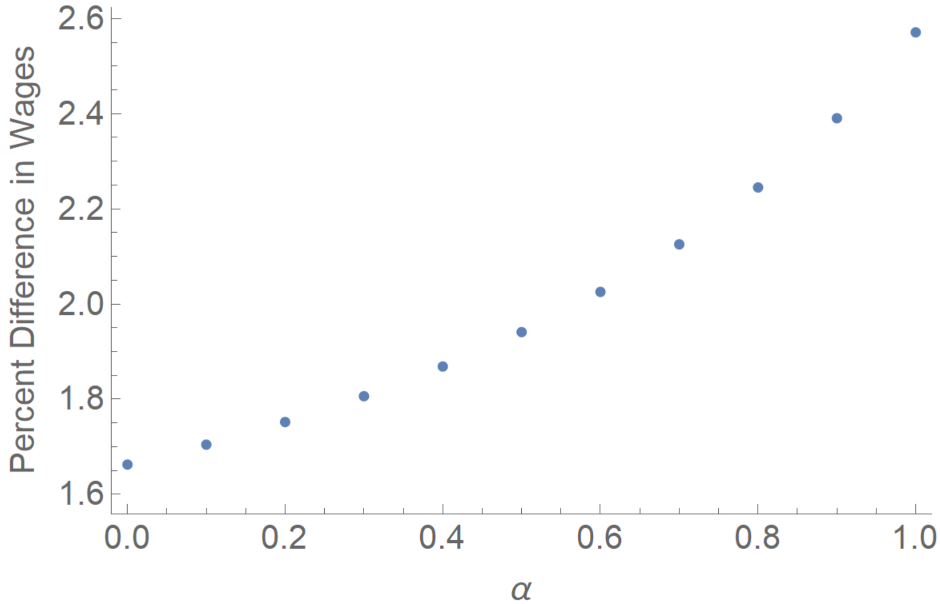
Because there are asymmetries in the exogenous parameters used to describe the high and low occupation types (including the cost of opening a vacancy, the matching technologies, and the flow utility of unemployment), we do not see workers sort perfectly into their most productive match even in the desegregated equilibrium. This has important consequences for who gains and who loses when switching from the segregated to desegregated equilibrium. The segregated equilibrium pushes white workers into more productive roles by making it less costly for them to obtain those roles, while for black workers it's the opposite–they're pushed into less productive roles. When moving to the desegregated equilibrium, white workers lose this advantage and black workers no longer have the same incentive to work in less productive occupations. Thus the desegregated equilibrium is beneficial for black workers, but harmful for white workers.

Figure 5: Wages and Output



So far I have assumed $\alpha = 0$. How do the calibration results differ if the true value of α is some positive number? Recalibrating the model to allow for $\alpha > 0$ generates slightly larger wage differentials between black and white workers. In this sense, 1.66% can be viewed as a lower bound for the wage differentials generated by this model. Figure 6 shows the model-generated wage differentials when recalibrated for different values of α.

Figure 6: Percent difference in black-white wages with $\alpha \in [0, 1]$



6.1 Efficiency, Stability, and Policy

I define social welfare W as aggregate output plus the flow value of unemployment net of vacancy creation costs.

$$W = \sum_r \tau^r \int_{x_j^r} e_H^r A_H x dx + \sum_r \tau^r a_L^r e_L^r A_L + \sum_r \sum_j z_j(\tau^r a_j^r u_j^r) - \sum_j v_j c_j$$

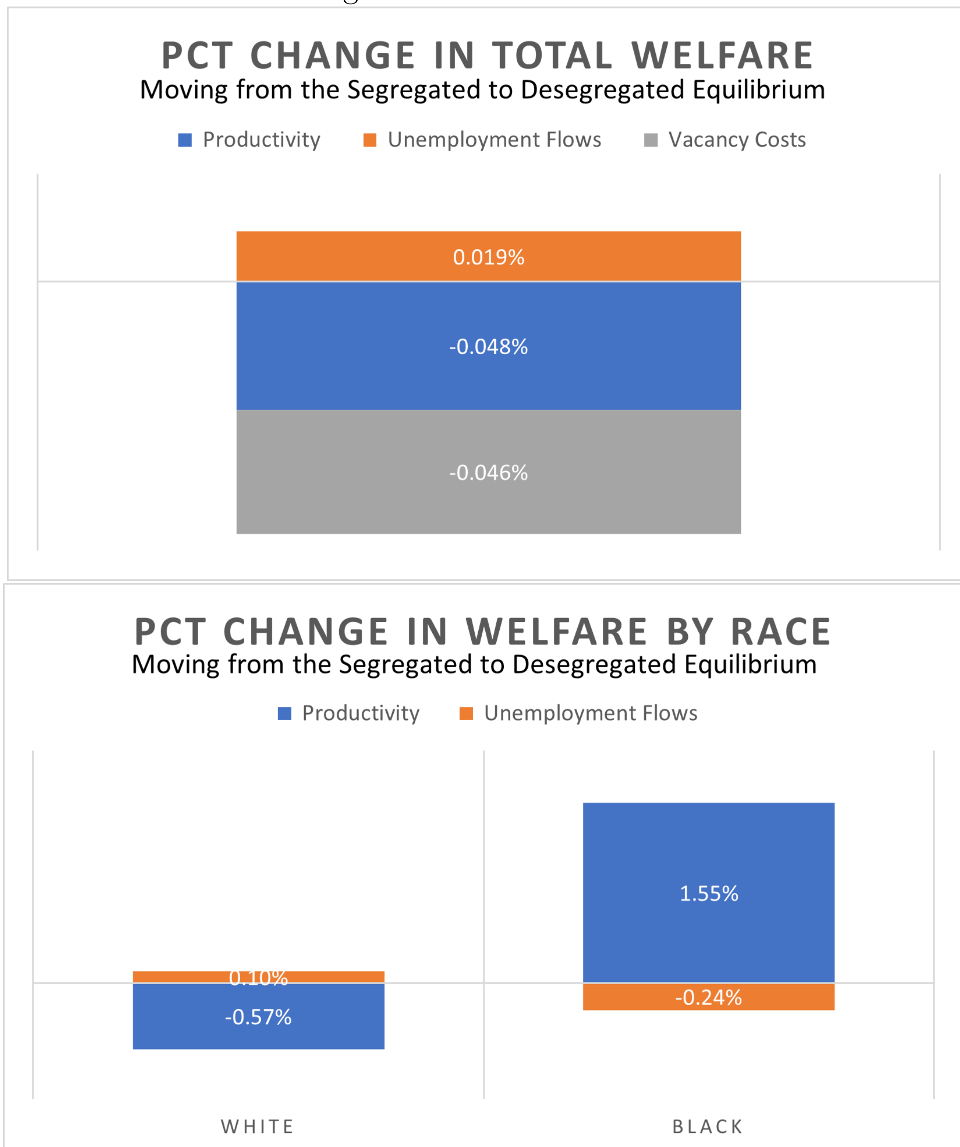
In a scenario with occupational segregation, some workers are sacrificing their productive advantage for the benefit of an increased likelihood of finding a job. This trade-off exists at the worker level and can also exist in the aggregate. Figure 7 shows the change in aggregate welfare when moving from the segregated to the desegregated equilibrium. It also shows the change separately for black and white workers, excluding any anaylsis of vacancy costs.

The results indicate that referral networks can create inequality while also enhancing aggregate welfare[12]. In the case here, referrals help the majority group (white workers) and harm the minority group (black workers). In aggregate, the gains for the majority group outweigh the losses for the minority group and lead to a measure of higher aggregate welfare in the segregated steady state relative to the desegregated steady state, although the difference is quite small.

The fall in productivity from the segregated to desegregated steady state is due in part to less employment among white workers in the high-type occupation. While more black workers are employed in the high-type occupation, increasing productivity for the black population, their minority status makes their gains relatively small in the aggregate. Beyond this, vacancy costs are higher, as well as the value of unemployment flows, because it is harder to form a match through the referral networks in the desegregated steady state. Integration diminishes individuals' employment prospects due to weaker network effects when there is a more even mix of occupations within an individuals' network.

[12]Galenianos (2021) uses a referral model to examine inequality as well as overall welfare. The effects of referrals on welfare are subtle. In this paper, referrals reduce welfare when workers face a different probability of forming a match despite having the same productivity. However, when worker heterogeneity is due to productivity differences, the network favors the more productive type and enhances welfare despite increasing inequality.

Figure 7: Welfare Results



Social inequalities demand policies addressing information gaps, network divisions, and leveraging feedback effects and social multipliers. While efforts to reduce homophily could be effective in reducing informational disadvantages for certain groups, it may not be necessary. The model's results show that once these groups are evenly distributed across occupations, a desegregated equilibrium is supported even in a world with substantial homophily.

However, when multiple equilibria exist, questions will arise about their stability. Dynamic processes select among equilibria, and it is informative to ask how we came to select a particular equilibrium, and how stability matters for the effectiveness of policies. In this model, we are in equilibrium when no worker can get higher utility from moving to another occupation. This decision is captured by the proportion of workers searching and working in each occupation[13][14]. The idea of stability is to examine the robustness of a set of equilibria to perturbations in the underlying game.

Define $\Delta U^r(x, a_H^W, a_H^B) = U_H^r(x) - U_L^r$ (note that U_j^r are a function of a_j^r through the matching function, which has been excluded from the notation thus far to reduce clutter). At x_r^*, in steady state $(\hat{a}_H^W, \hat{a}_H^B)$, we have $\Delta U^r(x_r^*, \hat{a}_H^W, \hat{a}_H^B) = 0$. I will define the stability concept using a standard myopic adjustment process of strategies[15]. Myopic adjustment dynamics have the property that at each instant the direction of movement in each population's strategy is weakly payoff increasing, given the current behavior of the other population(s)[16]. Myopic adjustment simply requires that utility increase along the adjustment path (holding fixed the play of other players).

Consider a dynamic system guided by the differential equation $\dot{a}_H^r = k\Delta U^r(x_r^*, a_H^W, a_H^B)$[17]. In the case of isolated equilibria, the Jacobian matrix informs us about their stability. A steady state is hyperbolic if $det(J[\Delta U^r(x)])$ has no eigenvalues with zero real parts. If the steady state is hyperbolic and the eigenvalues all have negative real parts, the steady state is called a sink. If the eigenvalues all have positive real parts, it's called a source. Otherwise,

[13]denoted by a_j^r, with $a_H^r + a_L^r = 1\ \forall r$ and $a_H^r = \int_{x \geq x_r^*} g(x)dx\ \forall r$.

[14]This is an equilibrium of pure strategies, where a_j^r s the measure of players in group r choosing pure strategy j.

[15]This is similar to Buhai and Leij (2023).

[16]See Swinkels (1992) for further discussion of myopic adjustment strategies.

[17]This is essentially defining the players response function. The player, defined by their ability and race, chooses their action (which occupation to search in) to maximize the expected payoff given their assesment of other players actions.

a hyperbolic steady state is called a saddle. A sink is asymptotically stable[18]. Define the Jacobian, evaluated in the steady state, as

$$J[\Delta U^r(x)] = \begin{bmatrix} \frac{\partial \Delta U^B(x_B^*, a_H^W, a_H^B)}{\partial a_H^B} & \frac{\partial \Delta U^B(x_B^*, a_H^W, a_H^B)}{\partial a_H^W} \\ \frac{\partial \Delta U^W(x_W^*, a_H^W, a_H^B)}{\partial a_H^W} & \frac{\partial \Delta U^W(x_W^*, a_H^W, a_H^B)}{\partial a_H^B} \end{bmatrix}$$

Examining both the selected segregated equilibrium and the hypothetical desegregated equilibrium, I find that both are hyperbolic, while the segregated is a sink (therefore stable) and the desegregated equilibrium is a saddle[19]. This is relevant when it comes to policy recommendations, indicating that policies with the goal of moving to and maintaining the desegregated equilibrium would need to be permanently maintained, otherwise society will naturally move to a segregated equilibrium. There is some empirical support for this in Kurtulus (2013), who uses Affirmative Actions bans at the state level to examine the impact of eliminating Affirmative Action policies on employment. She finds that there are deleterious effects on employment for minorities once the bans are set in place, indicating a lack of persistence in Affirmative Action policies. Myers (2007) focuses specifically on the Affirmative Action ban in California in 1996, and finds that employment among women and minorities dropped sharply, suggesting again a lack of persistence in Affirmative Action policies.

7 Conclusion

I've provided some empirical evidence for differences in the use of referral networks by race and occupation. I have proposed a search and match model where social interactions are an important component of the matching process. I have incorporated worker heterogeneity in ability and network composition, and allowed for minority and majority groups. This

[18]For a review of this material, see Fudenberg and Levine (1998).
[19]The third (segregated) equilibrium discussed in Appendix C is a source, therefore also unstable.

model is able to rationalize salient features of the data, including differences in the use of referral networks by race and differences in occupational choice by race. I find that racial homophily, or social segregation, can perpetuate occupational segregation in the steady state and give rise to inter-group inequality that aligns with empirically observed racial disparities. In particular, this mechanism alone can generate a 1.66% difference in wages and a 0.74 percentage point difference in employment, respectively accounting for 14 percent and 10 percent of the observed gaps for black and white non-college workers.

I also examine the welfare effects of a segregated versus a desegregated equilibrium under the same fundamentals, and find that aggregate welfare is higher in the segregated steady state. However, these gains accrue to white workers. Beyond this, I find that while a desegregated equilibrium is supported by the model, it is unstable. This has implications for the long-term impact of policies such as affirmative action. Once these policies are removed, society would likely move back towards a segregated equilibrium.

Since the empirical evidence for network use by race and occupation is still rather limited, these findings should be interpreted with some caution and future research is needed to confirm their robustness. Future theoretical work could also include endogenous on-the-job network formation to study the importance of business contacts, in particular for college-educated workers, as well as on-the-job search. Solving for a fully dynamic model would also allow for analysis of transition dynamics.

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9 Appendix

9.1 Appendix A

Occupation Categorization:

Table 7: Median & Mean Wages by SOC

SOC	Description	Median	Mean
L occupations:			
35	Food Preparation and Serving Related	10.41	11.12
45	Farming, Fishing, and Forestry	10.86	12.03
39	Personal Care and Service	11.45	12.71
37	Building and Grounds Cleaning and Maintenance	12.01	13.43
41	Sales and Related	12.26	13.76
31	Healthcare Support	12.94	14.07
25	Education, Training, and Library	13.63	15.88
43	Office and Administrative Support	15.61	16.80
53	Transportation and Material Moving	15.28	16.88
H occupations:			
51	Production	16.08	17.66
21	Community and Social Services	15.66	17.84
33	Protective Service	16.84	19.26
13	Business and Financial Operations	18.29	19.92
11	Management	18.09	20.48
23	Legal	18.52	20.75
27	Arts, Design, Entertainment, Sports, and Media	18.14	20.77
47	Construction and Extraction	19.78	22.02
19	Life, Physical, and Social Sciences	20.74	22.16
49	Installation, Maintenance, and Repair	21.43	22.61
29	Healthcare Practitioners and Technical	21.49	23.42
15	Computers and Mathematics	22.27	25.10
17	Architecture and Engineering	23.66	25.50

Data sources: CPS, 1995m1-2021m9, author's calculations

Occupational Segregation Decomposition. Each observation belongs to a joint distribution $F(j, z, r)$ of occupations $j \in \{H, L\}$, individual characteristics $z \in \{z_1, \ldots, z_k\}$

defined over domain Ω_k, and a dummy r indicating group membership $r \in \{B, W\}$. The joint distribution of occupations and attributes of each group is a conditional distribution $F(j, z|r)$. The discrete density function of occupations for each group, $f^i(j)$ can be expressed as the product of two conditional distributions:

$$f^i(j) \equiv f(j|r=i) = \int_z dF(j, z|r=i)dz = \int_z f(j|z, r=i) \cdot f(z|r=i)dz$$

where $i = 1$ for whites and $i = 0$ for blacks. Assuming the structure of occupations for blacks $(f(j|z, r=0))$ doesn't depend on the distribution of attributes, I can define a counterfactual distribution $f_z(j)$:

$$\begin{aligned} f_z(j) &= \int_z f(j|z, r=0) \cdot f(z|r=1)dz \\ &= \int_z f(j|z, r=0) \cdot \varphi_z \cdot f(z|r=0)dz \\ &= \int_z \varphi_z f(j, z|r=0)dz \end{aligned}$$

If blacks kept their own conditional probability of being in a given occupation, $f(j|z, r=0)$, but had the same characteristics of whites given their marginal distribution $f(z|r=1)$, then this is the density that would occur. This can be produced by properly re-weighting their original distribution.

Using Bayes' Theorem: $f(z|r=i) = \frac{Pr(r=i|z)Pr(z)}{Pr(r=i)}$. Then

$$\varphi_z = \frac{f(z|r=1)}{f(z|r=0)} = \frac{Pr(r=0)Pr(r=1|z)}{Pr(r=1)Pr(r=0|z)}$$

This is the unconditional probabilities of group membership (a constant) times the conditional probabilities that can be obtained by pooling the samples of black and white workers and estimating a logit model for the probability of being white conditional on z, i.e. $Pr(r = 1|z) = \frac{\exp(z\hat{\beta})}{1+\exp(z\hat{\beta})}$ where $\hat{\beta}$ is the vector of estimated coefficients.

Allow $S(j|z) \equiv S(f(j|r = 1), f_z(j))$ to denote the conditional segregation index. This is the amount of unexplained segregation that remains once I've controlled for observable characteristics.

SCE: different occupational groupings

How robust are results to different groupings of occupations? Unfortunately, there are not enough observations to examine each 2-digit SOC code separetely in the SCE. To see finer patterns, I can instead consider 3 different groups: low, medium, and high. Low includes the bottom seven SOC codes from Table 7, high the top seven SOC codes, and medium the eight middle SOC codes. Table 8 displays summary statistics for black and white workers by these 3 occupations.

While white workers usage of referrals is more evenly spread across occupation types, black workers have a clear pattern–they rely on referrals the most for the lowest-type occupations, and the least for the highest-type occupations. Note however that for black workers, I'm relying on a small number of observations and the standard errors can become quite large.

Results for the same logistic regression that was evaluated in section 3 for the two occupations are given in Table 9.

Table 8: Referral usage by race and occupation

	Black	White
Low	43.6	35.0
	(10.4)	(4.5)
Medium	25.1	31.9
	(5.7)	(2.7)
High	20.7	31.5
	(11.7)	(5.0)

Standard errors are shown in parenthesis.

Table 9: Logistic regression: referral usage by race and occupation

Med occupation	-0.834	-0.914
	(0.515)	(0.601)
High occupation	-1.088	-1.220
	(0.803)	(0.901)
White dummy	-0.361	-0.715
	(0.459)	(0.511)
Med Interaction	0.693	1.198*
	(0.566)	(0.659)
High Interaction	0.930	1.425
	(0.858)	(0.961)
State FE		x
Worker Characteristics		x
Psuedo R2	0.0052	0.1554
Obs	866	733

Robust standard errors are shown in parenthesis.
$*p < 0.1$; $**p < 0.05$

MCSUI: by city

Atlanta has 508 observations, 168 white. Los Angeles has 681 observations, 235 white. Boston has 424 observations, 206 white.

Racial breakdown of friend/family references (Atlanta):

		same race	other race (white/black)	other race (other)
white	(high occ)	1	0	0
	(low occ)	99.6	0.4	0
black	(high occ)	76.4	16.2	7.3
	(low occ)	1	0	0

Source: Multi-City Survey of Urban Inequality (MCSUI), author's calculations

Racial breakdown of friend/family references (Los Angeles):

		same race	other race (white/black)	other race (other)
white	(high occ)	87.0	4.3	8.6
	(low occ)	80.6	19.0	0.4
black	(high occ)	1	0	0
	(low occ)	95.6	2.0	2.4

Source: Multi-City Survey of Urban Inequality (MCSUI), author's calculations

Racial breakdown of friend/family references (Boston):

		same race	other race (white/black)	other race (other)
white	(high occ)	99.8	0.1	0.1
	(low occ)	79.5	19.2	1.2
black	(high occ)	48.1	51.7	0.1
	(low occ)	79.1	16.4	4.4

Source: Multi-City Survey of Urban Inequality (MCSUI), author's calculations

MCSUI: summary statistics and regression analysis: I am able to conduct the same analysis done for the SCE using the data available through the MCSUI instead. While the same patterns are seen for men in the MCSUI as were seen for everyone in the SCE, the data for women in the MCSUI does not follow the same patterns. Figures 8 and 9 report the raw summary statistics as well as the predicted results from the logistic regression reported separately for men and women from the MCSUI. For comparison, these same results are reported for the SCE by gender in Figures 10 and 11. Although both men and women follow the same general pattern in the SCE, the summary statistics in particular for men are starker.

Figure 8: Men: Referral usage by race and occupation (MCSUI)

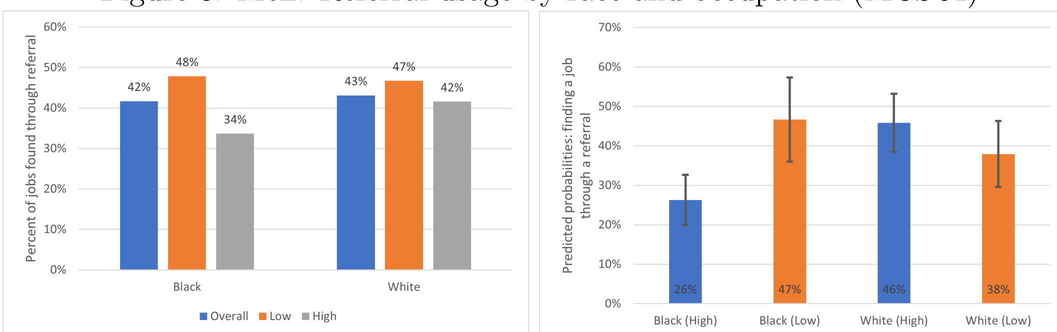


Figure 9: Women: Referral usage by race and occupation (MCSUI)

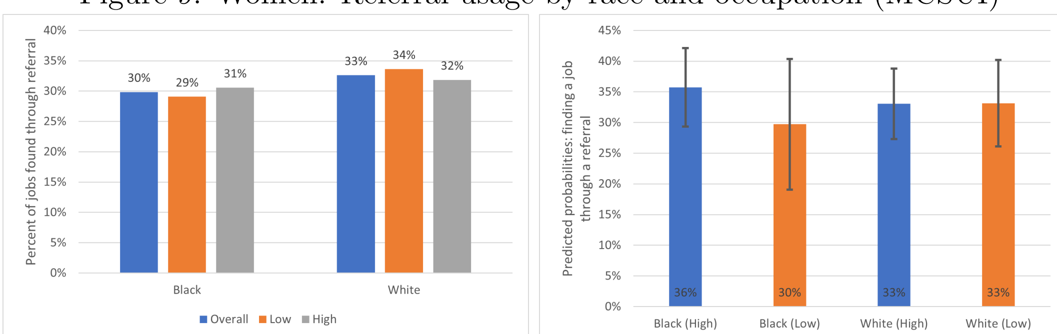


Figure 10: Men: Referral usage by race and occupation (SCE)

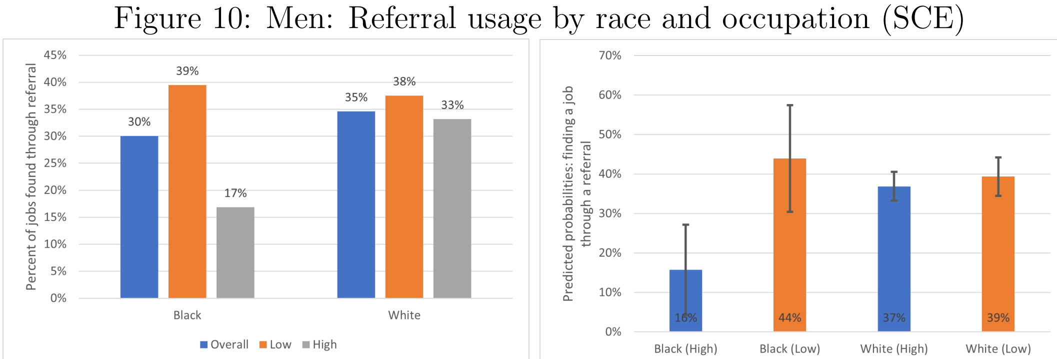
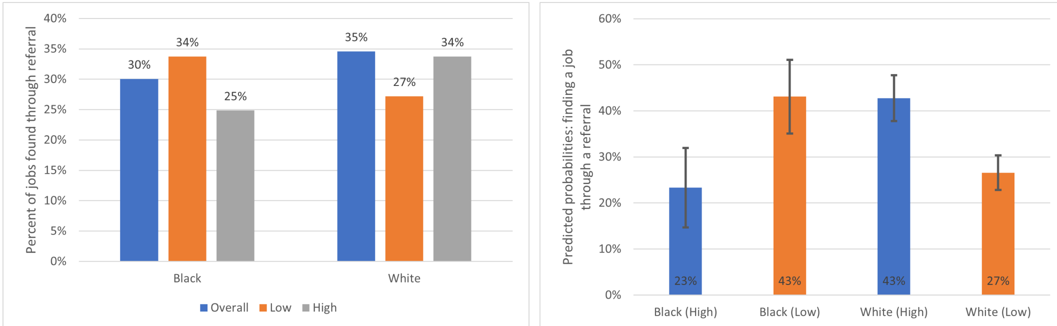


Figure 11: Women: Referral usage by race and occupation (SCE)



9.2 Appendix B

Proposition 4.1. The referral component of the job finding rate p_j^r is decreasing in unemployment u_j^r and $u_j^{\neg r}$ $\forall j, r$, therefore increasing in employment, i.e. $\frac{\partial p_j^r}{\partial e_j^r} > 0$ and $\frac{\partial p_j^r}{\partial e_j^{\neg r}} > 0$ $\forall j, r$. Assuming Γ_j is not too large, the referral job filling rate k_j^r is increasing in unemployment u_j^r and $u_j^{\neg r}$ $\forall j, r,$, therefore decreasing in employment, i.e. $\frac{\partial k_j^r}{\partial e_j^r} < 0$ and $\frac{\partial k_j^r}{\partial e_j^{\neg r}} < 0$ $\forall j, r$.

Proof. We have

$$p_j^r = \kappa_j (\bar{e}_j^r)^{\Gamma_j} = \kappa_j (\phi^r a_j^r e_j^r + (1 - \phi^r) a_j^{\neg r} e_j^{\neg r})^{\Gamma_j}$$

so that

$$\frac{\partial p_j^r}{\partial e_j^r} = \Gamma_j \kappa_j \phi^r a_j^r (e_j^{\neg r} a_j^{\neg r} (1 - \phi^r) + e_j^r a_j^r \phi^r)^{\Gamma_j - 1} > 0$$
$$\frac{\partial p_j^r}{\partial e_j^{\neg r}} = \Gamma_j \kappa_j (1 - \phi^r) a_j^{\neg r} (e_j^{\neg r} a_j^{\neg r} (1 - \phi^r) + e_j^r a_j^r \phi^r)^{\Gamma_j - 1} > 0$$

Now consider

$$k_j^r = \frac{u_j^r(\bar{e}_j^r)^{\Gamma_j}}{a_j^r e_j^r + a^{\neg r_j} e_j^{\neg r}} = \frac{(1-e_j^r)(\phi^r a_j^r e_j^r + (1-\phi^r)a^{\neg r_j} e_j^{\neg r})^{\Gamma_j}}{a_j^r e_j^r + a^{\neg r_j} e_j^{\neg r}}$$

giving us

$$\begin{aligned}
\frac{\partial k_j^r}{\partial e_j^r} = & -\overbrace{\left(\frac{(e_j^{\neg r}(1-\phi^r)a^{\neg r_j} + e_j^r \phi^r a_j^r)^{\Gamma_j - 1}}{(e_j^{\neg r} a^{\neg r_j} + e_j^r a_j^r)^2}\right)}^{-} \\
& * \Big(\overbrace{(e_j^{\neg r})^2 (a_j^{\neg r})^2 (1-\phi^r)}^{+} \\
& + e_j^{\neg r} a_j^r a_j^{\neg r}(1-(1-e_j^r)(1+\Gamma_j)\phi^r) + e_j^r \phi^r (a_j^r)^2 (1-(1-e_j^r)\Gamma_j)\Big) < 0
\end{aligned}$$

In order for the final line to have a positive value and guarantee that the partial derivative is negative, we need $(1-e_j^r)\Gamma_j < 1$ and $(1-e_j^r)(1+\Gamma_j)\phi^r < 1$, i.e. Γ_j cannot be too large.

Proposition 4.2. The aggregate matching function exhibits decreasing returns to scale.

Proof. Here I multiply $u_j^r\ \forall r$ and the occupation-specific vacancy rate v_j by a factor $\omega > 1$. Then

$$\begin{aligned}
& \theta_j(\omega v_j)^{\eta}(\omega \tilde{u}_j)^{(1-\eta)} + \omega u_j^r \kappa_j (\phi^r a_j^r (1-\omega u_j^r) + (1-\phi^r)a^{\neg r_j}(1-\omega u_j^{\neg r}))^{\Gamma_j} \\
& \quad = \omega M_j + \omega u_j^r \kappa_j (\phi^r a_j^r (1-\omega u_j^r) + (1-\phi^r) a_j^{\neg r}(1-\omega u_j^{\neg r}))^{\Gamma_j} \\
& \quad = \omega M_j + \omega u_j^r \kappa_j (\phi^r a_j^r + (1-\phi^r) a_j^{\neg r} - \omega \bar{u}_j^r)^{\Gamma_j} \\
& \quad = \omega M_j + \omega P_j \Big(\frac{\phi^r a_j^r + (1-\phi^r) a_j^{\neg r} - \omega \bar{u}_j^r}{\bar{e}_j^r}\Big)^{\Gamma_j}
\end{aligned}$$

It is necessarily the case that $\phi^r a_j^r + (1-\phi^r)a_j^{\neg r} - \omega\bar{u}_j^r < \bar{e}_j^r$. Written differently:

$$\phi^r a_j^r + (1-\phi^r)a_j^{\neg r} - \omega\bar{u}_j^r < \phi^r a_j^r + (1-\phi^r)a_j^{\neg r} - \bar{u}_j^r$$

$$\omega\bar{u}_j^r > \bar{u}_j^r$$

$$\omega > 1$$

Since $\left(\frac{\phi^r a_j^r + (1-\phi^r)a_j^{\neg r} - \omega\bar{u}_j^r}{\bar{e}_j^r}\right)^{\Gamma_j} < 1$, the function exhibits decreasing returns to scale.

9.3 Appendix C

Equal population sizes: I allow relative population sizes to differ in the baseline model. However, I do not need a majority/minority group dynamic to generate the patterns seen in the data. Here I show a version of the results where $\tau^r = 0.50$ and $\gamma^r = 0.90\ \forall r$. I will maintain the same set of targeted moments in this hypothetical world of equal population sizes, seen in Table 11.

Table 10: Parameter values: equal population sizes

Parameter	Value
A_H, A_L	0.727, 1.324
z_H, z_L	-0.049, -0.018
θ_H, θ_L	0.041, 0.041
κ_H, κ_L	0.537, 0.110
Γ_H, Γ_L	2.579, 1.249
σ_x	0.420

Table 11: Model Fit and Desegregated Equilibrium: equal population sizes

	Data	Targeted	Segregated	Desegregated
% B employed in H	37%	Y	37%	45%
% W employed in H	48%	Y	48%	45%
job-finding rate (B)	0.10	Y	0.10	.0998
job-finding rate (W)	0.10	Y	0.10	.0998
mean wages (H)	1.39	Y	1.39	1.395
90th percentile wages (H)	2.22	Y	2.22	2.26
mean wages (L)	1.00	Y	1.00	1.001
% jobs from network:				
Black (L):	35%	Y	35%	32%
Black (H):	22%	Y	22%	30%
White (L):	32%	Y	32%	32%
White (H):	33%	Y	33%	30%

Data sources: CPS, SEC, author's calculations

Figure 12: Wages and Productivity: equal population sizes

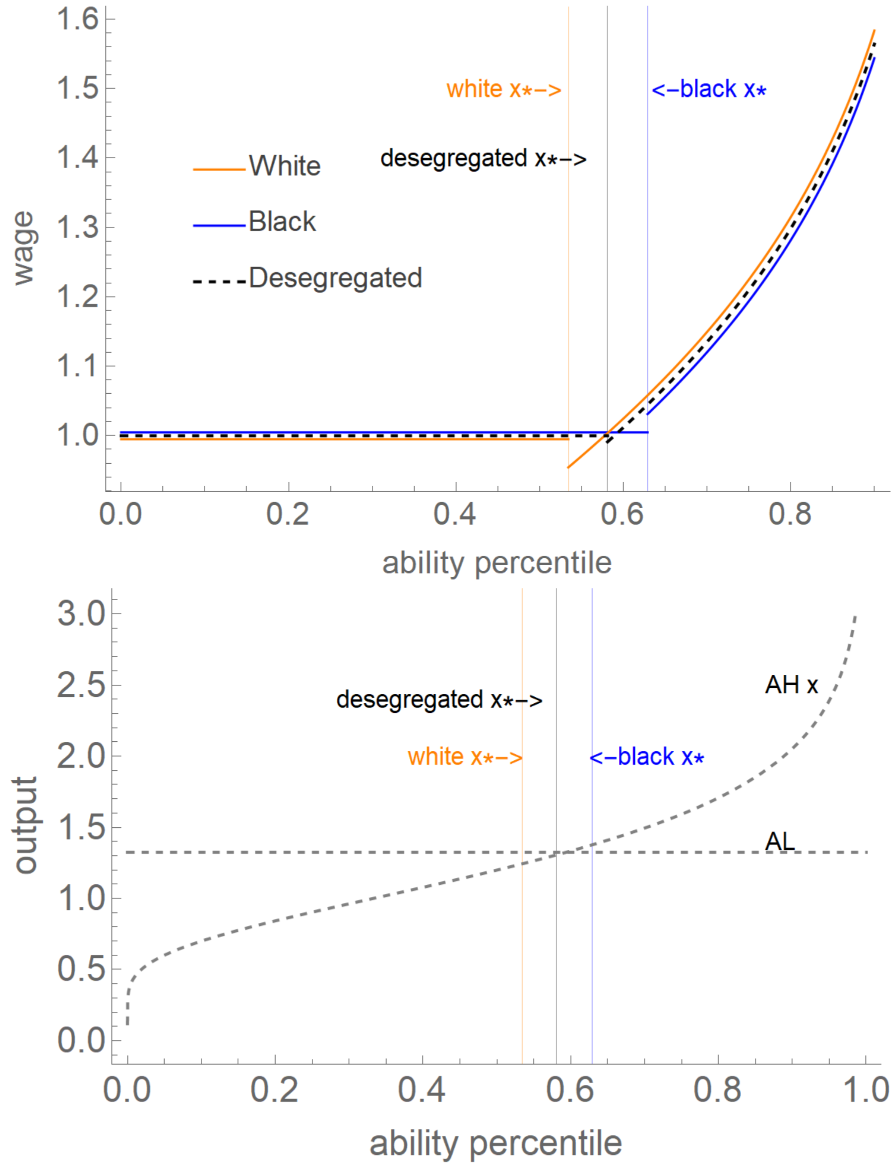
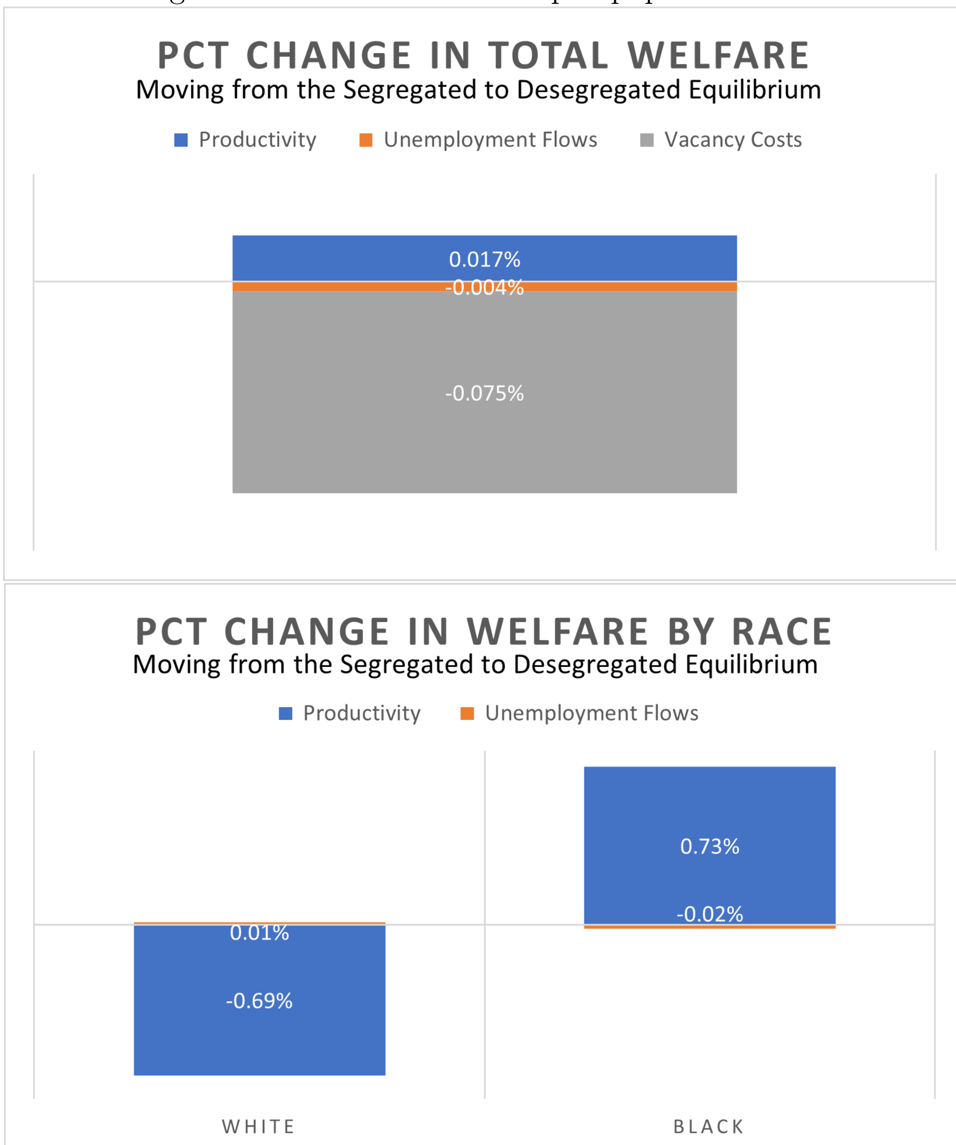


Figure 13: Welfare Results: equal population sizes



In contrast to the results where the population sizes differ by race, both black and white workers move to more suitable occupations in the desegregated equilibrium. Also unlike the previous results, in the aggregate the gains to black workers cancel out the losses to white workers when moving from the segregated to the desegregated equilibrium. However, once vacancy costs are accounted for aggregate welfare still decreases. It is more difficult to match workers and firms together when they worker's networks contain fewer contacts in the same occupation as them, which is the case in the desegregated equilibrium. Put differently, the positive network externality generated by searching in the same occupation as your peers dominates the negative externality generated by the misallocation of ability levels for black workers in the segregated equilibrium.

The (other) segregated equilibrum: While the focus is on the observed segregated and theoretical desegregated equilibria, it's interesting to note that there is one more segregate equilibrium supported by the model and the main set of parameter values, where black workers are overemployed in H occupations and white workers are overemployed in L occupations. Results for this equilibrium are available in Table 12 and, for convenience, the results for the other two equilibria are reported again as well. For employment and wage differences, black workers have an employment rate 16 percentage points higher than white workers, and a wage rate that is 9% higher on average. Note also that this equilibrium is a source (and therefore unstable).

Table 12: Three equilibria

	Data	Segregated (selected)	Desegregated	Segregated (other)
% B employed in H	37%	37%	45%	87%
% W employed in H	48%	48%	45%	32%
job-finding rate (B)	0.10	0.10	.0998	.34
job-finding rate (W)	0.10	0.10	.0998	.098
mean wages (H)	1.39	1.39	1.395	1.42
90th percentile wages (H)	2.22	2.22	2.26	1.96
mean wages (L)	1.00	1.00	1.001	1.07
% jobs from network:				
Black (L):	35%	35%	32%	3%
Black (H):	22%	22%	30%	85%
White (L):	32%	32%	32%	34%
White (H):	33%	33%	30%	22%

Data sources: CPS, SEC, author's calculations